KZH-Fold: Sublinear accumulation

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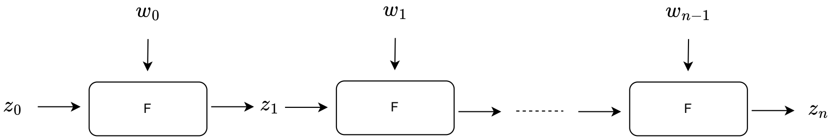
Background

What is Incremental Verifiable Computation (IVC)? [Val08]

- **IVC**:

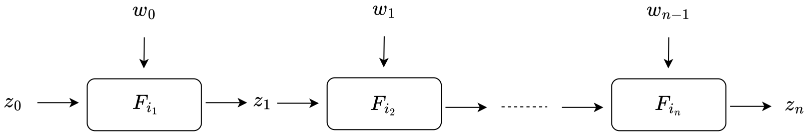


- **PCD**: Generalizes IVC to DAGs

Non-Uniform IVC/PCD?

Step function F can be one of multiple predefined instructions F_1, ..., F_k

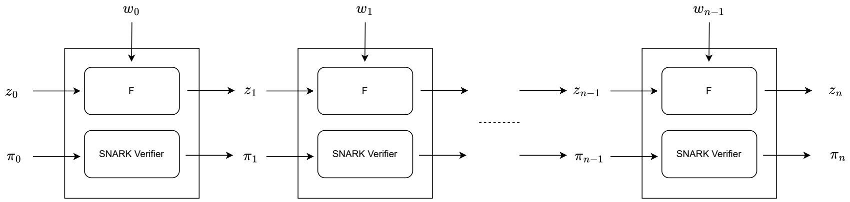
- Motivation: Verifiable CPU



Traditional IVC construction? [BCC+13]

IVC $\implies$ Augmented circuit:

1. Subcircuit F
2. SNARK verifier subcircuit (recursive overhead)



IVC Construction from Accumulation Schemes [Halo]

X SNARK verifier in circuit is expensive!

Observation. Defer checking all proof $\implies$ accumulation schemes

Accumulation Scheme

Accumulation Scheme [BCMS21;KST21;KP22; BC23]

- Let $\mathcal{L}_\pi$ and $\mathcal{L}_{acc}$ be two NP languages.
- An accumulation:

$$acc \in \mathcal{L}_{acc},\ \pi \in \mathcal{L}_\pi \overset{\text{accumulate}}{\Longrightarrow} acc' \in \mathcal{L}_{acc}$$

satisfies *completeness* and *knowledge soundness*.

Accumulation Scheme [BCMS21;KST21;KP22; BC23]

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- An accumulation:

$$acc \in \mathcal{L}_{acc},\, \pi \in \mathcal{L}_\pi \overset{\text{accumulate}}{\Longrightarrow} acc' \in \mathcal{L}_{acc}$$

Completeness: acc and π are satisfied $\iff$ acc' is satisfied.

- Decider = Satisfiability function of $\mathcal{L}_{acc}$

Accumulation Scheme [BCMS21;KST21;KP22; BC23]

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- An accumulation:

$$acc \in \mathcal{L}_{acc},\ \pi \in \mathcal{L}_\pi \overset{\text{accumulate}}{\Longrightarrow} acc' \in \mathcal{L}_{acc}$$

Knowledge soundness: witness for acc' $\implies$ witness for acc and π.

Sublinear Accumulation

Sublinearity:

- acc $\in \mathcal{L}_{acc}$, $\pi \in \mathcal{L}_{\pi} \implies |acc| \in o(|\pi|)$
- Decider time $<$ Verification of π

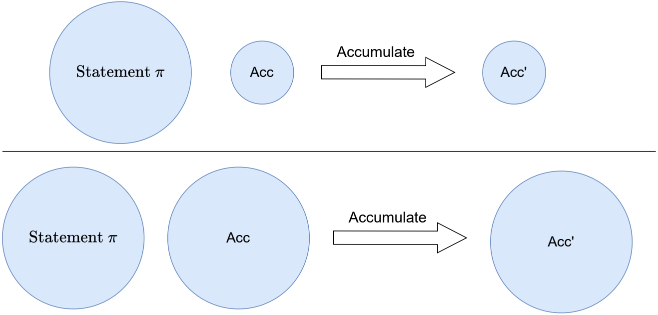
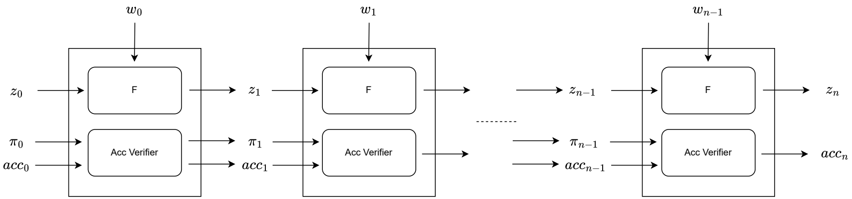


Figure: Sublinear vs Linear-Sized Accumulation Scheme

Build IVC from Accumulation Scheme

How to Use Accumulation to Build IVC?



$\pi_1, \pi_2, \ldots, \pi_n \implies$ Accumulate them into acc_n and check acc_n.

BCLMS Compiler: Accumulation for NARK $\implies$ IVC

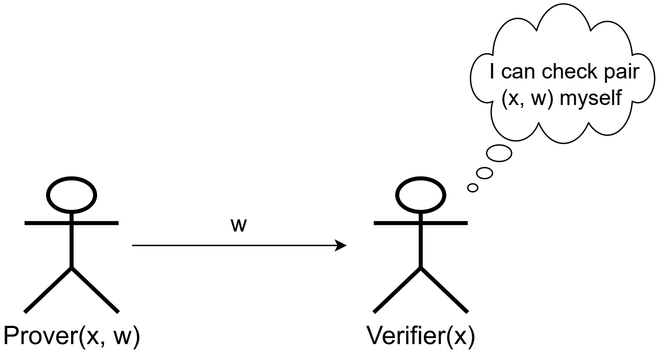
BCLMS Compiler: NARK + NARK Verifier Accumulation $\implies$ IVC

- IVC proof $=$ $|\mathsf{acc}|$
- IVC verifier $=$ $\mathsf{Decider}_{\mathsf{acc}}$
- IVC prover $=$ $\mathsf{Prover}_{\mathsf{NARK}}$ (for augmented circuit) $+$ $\mathsf{Prover}_{\mathsf{acc}}$

Sublinear accumulator $\implies$ IVC/PCD with sublinear proofs/decider

Example of Nova

Trivial Nark



Nova (Accumulation for Trivial R1CS NARK)

Nova accumulator for (relaxed) R1CS: (n =instance+witness size of R1CS)

- Accumulator size: $O(n)$
- Accumulator prover time: $O(n)$
- Accumulator decider time: $O(n)$
- Verifier circuit: $O(1)$, 2 or 3 scalar multiplications

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Nova IVC via BCLMS compiler:

- IVC proof size: $O(n)$
- IVC prover time: $O(n)$
- IVC decider time: $O(n)$

Nova Compression Phase

Consider a function (circuit) F to be $1M$ constraints $\implies$

- accumulator size: $80MB$
- decider time: $4s$

$\implies$ Impractical distributed proving.

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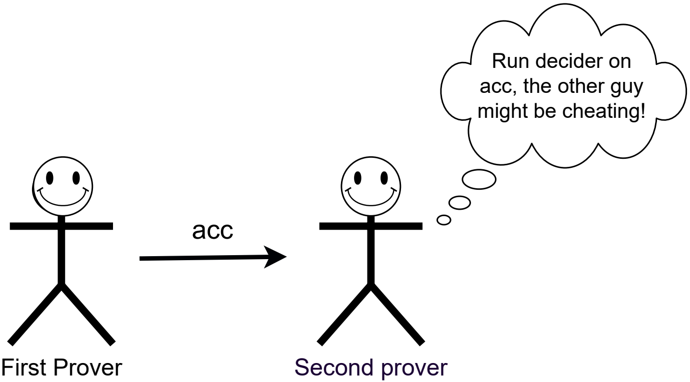
MicroNova/Nova solution: Run a zkSNARK at the last step of IVC $\implies$ reduces proof size ($\approx 1KB$)

X $24x$ prover overhead

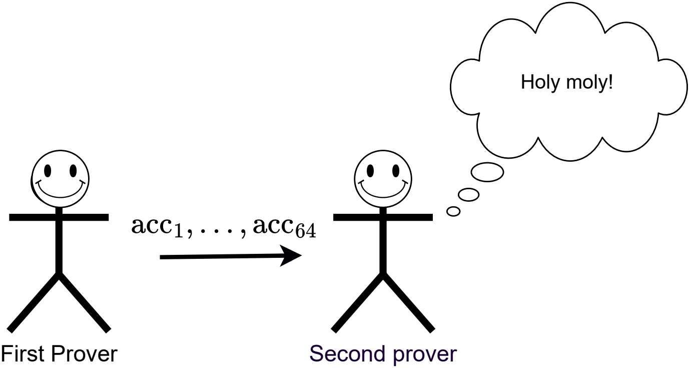
X No longer incremental.

Motivations of Paper

Motivation: Distributed Proving



Motivation: Distributed Proving of zkVM with SuperNova



- N-IVC/N-PCD with SuperNova $\implies$ One accumulator per instruction

Motivation For Sublinear Decider

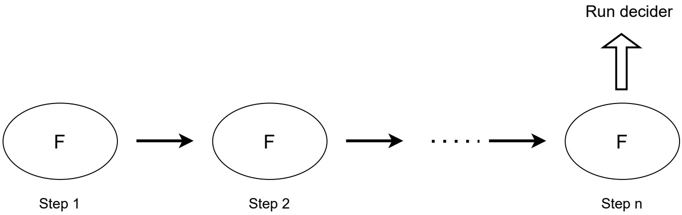


Figure: IVC when proof of the last step is needed.

Motivation: IVC When All Steps Matter!

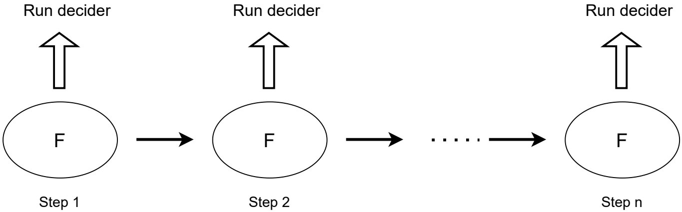


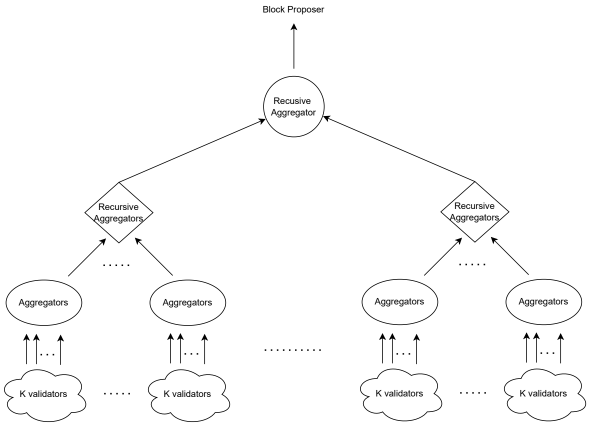
Figure: IVC when proof of all steps are needed.

Applications:

- Light clients
- Verifiable key directory
- Succinct blockchain

Motivation: Distributed signature aggregation (1)

- Ethereum finality time: 15min $\implies$ dominated by signature aggregation computation



Motivation: Distributed signature aggregation (2)

Observations:

- Computation of aggregators:
  - BLS aggregation
  - Union of bit vectors(mini PIOP)
  - Recomputing aggregate public key
- Signature aggregation is a layered tree $\implies$ model as PCD
- Communication is P2P $\implies$ we require low communication

Motivation: Distributed signature aggregation (3)

Signature Aggregation Based on PCD with sublinear:

- Improve verifier time and communication by 5x
- 5x improvement in finality time

Contribution

Contributions

- **KZH, PCS with Sublinear Opening**
- **Sublinear Accumulator Based on KZH**
  - IVC/PCD with sublinear proof and verifier.
  - Signature aggregation for Ethereum via PCD.
- **New Approach to N-IVC / N-PCD, First efficient N-PCD scheme based on any polynomial accumulation**

See numbers for KZH-fold

Scheme	**Prover**	**Verifier**	$\lvert \text{Acc} \rvert$	**# Constraints**
Spartan+KZH-fold	16.5 s	135 ms	37 KB	$2^{21} \approx 2097k$
Nova	4.8 s	5.6 s	80.8 MB	1185k

Table: Spartan+KZH-Fold vs Nova for Circuit With 2000 Poseidon Hashes

Starting Point

Goal

- IVC with sublinear proof/decider $\iff$ Sublinear accumulation.
- PCS with sublinear predicate $\implies$ Sublinear accumulation.

Goal: PCS with following properties:

- Homomorphic
- Constant size commitment
- Sublinear opening size
- Algebraic and low degree verifier checks
- No degree 2 pairing

$$\sum_i e(P_i, g_i) = \sum_j e(P_i', g_i') \implies g_i, g_i' \text{ are fixed.}$$

KZH and Its Accumulation Scheme: KZH-Fold

Hyrax:

+ PCS with square root opening size $\implies$ square root accumulator size
X Commitment $= O(\sqrt{n})$ group elements $\implies$ High recursive overhead

Starting point: Hyrax

Given a Pedersen setup $(g_1, g_2, \ldots, g_m)$, commit to rows of the coefficient matrix:

$$\begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1m} \\ t_{21} & t_{22} & \cdots & t_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ t_{k1} & t_{k2} & \cdots & t_{km} \end{bmatrix} \begin{matrix} \Longrightarrow & c_1 \\ \Longrightarrow & c_2 \\ \vdots & \\ \Longrightarrow & c_k \end{matrix}$$

Commitment size is $k = O(\sqrt{n}) \Rightarrow$ homomorphism/acc verifier is $\mathrm{O}(\sqrt{n})$

KZH: Commitment Phase

To commit to the matrix of evaluation points via KZH:

1. C: commit to the whole matrix in $1\mathbb{G}$ element, using a universal-SRS.
2. $\{D_i\}$: Commit to each row using $g_1, \ldots, g_m$.

$$\underbrace{\begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1m} \\ t_{21} & t_{22} & \cdots & t_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ t_{k1} & t_{k2} & \cdots & t_{km} \end{bmatrix}}_{C} \begin{array}{l} \Longrightarrow D_1 \\ \Longrightarrow D_2 \\ \quad\ \vdots \\ \Longrightarrow D_k \end{array}$$

Prove via pairing that C and $\{D_i\}$ correspond to the same matrix. Uses SRS.

KZH: Opening and Verification

- To open, send $\{D_i\}$ + Hyrax opening
- To verify an opening with respect to C and the opening:
  1. Prove that C and $\{D_i\}$ correspond to the same set of evaluation points.
  2. Run hyrax verification.

KZH properties

For a multilinear polynomial $f(\vec{X})$ with on boolean hypercube of size ℓ:

- commitment time $= O(\ell)$ group operations.
- Proof size $= \sqrt{\ell}\ \mathbb{G}_1 + \sqrt{\ell}\ \mathbb{F}$
- Opening time $= \ell\ \mathbb{F}$ [1]
- Verifier time $= \sqrt{\ell}$ pairing $+$ MSM$(2\sqrt{\ell}) + \sqrt{\ell}\ \mathbb{F}$.

[1] Dominated by polynomial evaluation.

KZH-k

We extend KZH matrix construction to tensors of higher degree $\implies$ Lower verifier cost at the cost of higher opening.

- Commitment cost: $O(n)$
- Opening cost: $O(n^{1/2})$ via pre-processing
- Verifier cost: $O(k \cdot n^{1/k})$

KZH-fold

The verifier function for KZH$=$ degree 2 scalar multiplications $+$ degree 1 pairing check $\overset{\text{Protostar compiler}}{\Longrightarrow}$

- Accumulator size. $O(\ell^{\frac{1}{2}})$
- Decider complexity. $O(\ell^{\frac{1}{2}})$
- Prover complexity. $O(\ell)$
- Verifier complexity. $3-4$ $\mathbb{G}_1$ scalar multiplications $+$ $O(1)$ $\mathbb{F}$

KZH-k fold:

- $O(k \cdot n^{1/k})$ decider and accumulator size
- $k+1$ scalar multiplication in recursive circuit.

Compariosn of kzh-fold and other schemes

Scheme	Recursive Overhead	Decider	—acc—
Nova	2 group ops	MSM(n)	$O(n)$
KZH2-fold	3 group ops	$n^{\frac{1}{2}}$ P	$O(n^{\frac{1}{2}})$
KZH-k fold	$k+1$ group ops	$k \cdot n^{\frac{1}{k}}$ P	$O(k \cdot n^{\frac{1}{k}})$
Halo	$O(\log n)$ group ops	MSM(n)	$O(\log n)$

IVC/PCD for R1CS From PCS Accumulation

Accumulation for NP from PCS Accumulation

$$\text{NP} \overset{\text{PIOP}}{\Longrightarrow} \text{Polynomial checks} \overset{\text{PCS Acc}}{\Longrightarrow} \text{Accumulate polynomial checks}$$

$$\text{R1CS} \overset{\text{Spartan}}{\Longrightarrow} \text{Polynomial checks} \overset{\text{PCS Acc}}{\Longrightarrow} \text{Accumulate polynomial checks}$$

Accumulation for R1CS from PCS Accumulation

R1CS $\overset{\text{Spartan}}{\Longrightarrow}$ witness polynomial $w(\cdot)$ + matrices A, B, C evaluations

- $w(\cdot) \implies$ interpolate $w(\cdot)$ through a PCS and accumulate.
- Matrices A, B, C can be evaluated as KZH, i.e. KZH works for sparse matrices too.

Better way $\implies$ accumulate following relation directly:

$$\mathcal{R}_{\tilde{A}} = \{(r_x \in \mathbb{F}^{\mu_n}, r_y \in \mathbb{F}^{\mu_m}, z \in \mathbb{F}) : \tilde{A}(r_x, r_y) = z\}$$

Accumulate A, B, C matrix evaluations

- Prover cost: $\log n$ evaluation of $\tilde{A}$ (out of circuit)
- Verifier cost (circuit size): $O(\log n)\mathbb{F}$
- Proof size: $O(\log n)\mathbb{F}$

Accumulation for R1CS

R1CS $\overset{\text{Spartan}}{\Longrightarrow}$ witness polynomial $w(\cdot)$ + matrices A, B, C evaluations

- $w(\cdot) \implies$ interpolate $w(\cdot)$ as PCS (KZH) and accumulate with PCS accumulator (KZH-fold).
- Accumulate matrix evaluation of A, B, C directly.

IVC from PCS accumulation

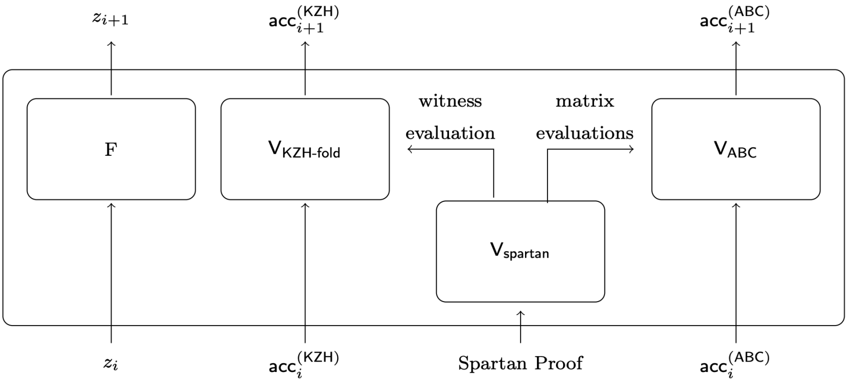


Figure: Augmented circuit initiated with KZH-fold

Non-Uniform PCD from PCS Accumulation

Comparison of Approaches to N-PCD

Scheme	Prover Time	Verifier Time	Witness Size
SuperNova	$O(\sum \|F_i\|)\mathbb{G}$	$O(\sum \|F_i\|)\mathbb{G}$	$O(\sum_i \|F_i\|)$
Protostar	$O(\sum \|F_i\|)\mathbb{G}$	$O(\sum \|F_i\|)\mathbb{G}$	$O(\sum_i \|F_i\|)$
KiloNova	$O(\sum \|F_i\|)\mathbb{G}$	$O(\sum \|F_i\|)\mathbb{G}$	$O(\sum_i \|F_i\|)$
Spartan+PA	$\mathcal{P}_{\text{acc}}(\max_i \|F_i\|) + \sum_i \log \|F_i\|$	$\mathcal{D}_{\text{acc}}(\max_i \|F_i\|) + O(\sum_i \|F_i\|)\mathbb{F}$	$O(\|acc\| + \sum \log_i \|F_i\|)$

PA=KZH-Fold:

- Prover time: $O(\max_i |F_i|) + \sum_i \log |F_i|$
- Verifier time: $O(\sqrt{\max_i |F_i|}) + O(\sum_i |F_i|)\mathbb{F}$
- Witness size: $\sqrt{\max_i |F_i|} + \sum \log_i |F_i|$

Non-Uniform PCD from PCS Accumulation

High-level idea:

- PCS accumulation $\implies$ more flexible than circuit accumulation
- Polynomials of different degrees can be accumulated.

Comparison to Supernova:

- directly accumulates circuit $\implies$ one running accumulator for each instruction.

Non-Uniform IVC from PCS Accumulation (1)

$F_i \overset{\text{Spartan}}{\Longrightarrow} \omega_i(\cdot)$ and matrix evaluation of A_i, B_i and C_i.

- $\omega_i(\cdot) \implies$
  1. Consider a running polynomial of degree $\deg(\omega_i) < D$.
  2. Accumulate $\omega_i(\cdot)$ with this running PCS accumulator.
- matrix evaluations of A_i, B_i and C_i
  - Similar strategy to SuperNova

Key to efficiency: Matrix evaluations scale logarithmically with the size of the original circuit.

THNAKS!